A Dynamic Programming Model for Structuring Mortgage Backed Securities

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Abstract— **This paper presents a Dynamic Programming model that reduces the cost of issuing a Mortgage-Backed Security by changing the structure of the security issued. The implementation is built on the jMDP framework, which solves Markov Decision Problems. The model was implemented in a real life instance, using the original data of the seventh issuance of the Colombian securitizing firm, Titularizadora Colombiana (a Fannie Mae alike), which allowed us to determine the different levels of improvement attained by the model.**

I. INTRODUCTION

Mortgage Securitization is a financial instrument widely used in the United States as a means to finance, give liquidity and deepen the mortgage sector. In Colombia this financial instrument was recently introduced and its continuous improvement is one of the key ways to boost the development of the construction and mortgage sectors, key elements in the development of an emergent economy.

One way to enhance the securitization process is by understanding its different stages and the steps that have to be taken in order to put an issuance in the capital market. This project focuses on the structuring process of the Mortgage-Backed Securities (MBS), that is, in the decisions taken to define the form that the security will have in the moment the issuance comes out to the market.

We relied on a dynamic programming approach [1] to improve the structuring process of the securities, given that the decision making of the process has to be done in a sequential manner. The objective underlying the implementation of the dynamic programming model is to reduce the costs of issuing a Mortgage-Backed Security by minimizing the present value of the payments directed to the bondholders.

The computational implementation of the model is an extension of the jMDP program[2], an object oriented framework to model and solve Markov Decision Problems, and in particular for this project, Dynamic Programming Models, programmed in Java.

The main results can be summarized by showing that the application of this model to the seventh issuance of Titularizadora Colombiana, generated a change in the structure of the MBS, that translated in a reduction of the cost of the issuance ranging from 1.19% to 9.48%, that is a reduction of the cost between $1 to $15 million dollars depending on the structure chosen. The dynamic programming approach and its implementation in a real life instance proved to be efficient and useful.

The remainder of this paper is organized as follows. In Section II we give a brief description of what a Mortgage-Backed Security is, and of the current structure that is used to issue the securities out to the Colombian Capital Market. In Section III we describe the Dynamic Programming (DP) tool used to model and solve the core problem and introduce the jMDP framework. Section IV shows the summary of the outcomes and results obtained by the development and implementation of the DP model and in Section V we present the concluding remarks and further work.

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II. MORTGAGE-BACKED SECURITIZATION

A. Definitions

Securitization is a financial instrument which is based on the idea of receiving an amount of money today, and has as guarantee the future cash flows produced by the investment of that money. The guarantee is known as the underlying asset, and the investors who give out the money today and receive the future cash flows are called the bondholders [4].

In particular, Mortgage securitization can be understood as the process through which a securitizing company buys mortgages from the banks, packs those mortgages into one pool of assets, and issues securities in the capital market, having as underlying asset the pool of mortgages. In Colombia, the only securitizing company is called Titularizadora Colombiana, and the securities they issue are called TIPS.

B. Current Structure of MBSs (TIPS) in Colombia

The Mortgage-Backed Securities (TIPS) issued in Colombia have a rigid and predetermined structure that is composed of two classes of securities divided in four tranches whose payments to the bondholders are made sequentially. A tranche is the resultant of splitting the security into other smaller securities with shorter expiration dates, and specific characteristics regarding interest rate, duration and risk credit rating.

TIPS class A: This class is divided into three tranches each one with a different duration, interest rate and a risk rating. TIPS A_1 have a five years duration, the lowest interest rate and a AAA rating; TIPS A_2 have a 10 years duration and a AA rating; and TIPS A_3 have a 15 years duration and an A rating.

TIPS class B: This class does not have an inner division, acts as a single tranche with 15 year duration, risk rating A or lower, and the highest interest rate paid to the bondholders.

The main characteristic of the current structure of the securities is that the payments to the bondholders are made in a sequential manner, known as a waterfall payment structure. It means that the payments of the tranche A_2 do not begin until the last payment to the tranche A_1 is made, payments of the tranche A_3 do not begin until the last payment to the tranche A_2 is made and so on.

This project focuses on this particular problem, the rigid and standardized structure, to search for a flexible, better and cost efficient structure. The hypothesis underlying the development of this project is that by changing the rigid structured used in Colombia to define the Mortgage-Backed Securities, and by finding a new, flexible structure that accounts for the inner characteristics of the pool of mortgages being securitized, the new structure would be less expensive, and cost efficient in the way that it would still be able to fulfill its liabilities to the bondholders and will not fall in default. The development of the new structure will be explained in the following section.

III. Dynamic Programming and jMDP

A. Dynamic Programming

Dynamic Programming is a term used to refer to both, a model methodology as well as the different approaches taken when solving problems where decision making is made sequentially [3]. This is best understood as the situations where the information that one has in a particular moment of time is useful to take immediate decisions, but once that decision is taken, new information will come out to broaden the space of possible decisions to make.

The dynamic programming tools help us determine optimal strategies that will take into account both present and future information as well as the space of possible decisions to make.

Dynamic programming models can be classified according to its inner characteristics, such as discrete or continuous space, finite or infinite time horizon and deterministic or stochastic decisions. When addressing the core problem of this project, we will refer particularly to a dynamic programming model with a discrete space of events, finite time horizon and deterministic decisions.

The development and solution methodology of a dynamic programming model are based on Bellman's *Principle of Optimality*. This principle states, in a broad definition, that in order to achieve general optimality in a specific model that follows a process of sequential decision making, all the remaining decisions after reaching a particular *state* must be optimal with respect to that state. That is, if a solution strategy follows a suboptimal decision in any intermediate step of the process, the problem as a whole will not be optimal.

We will now explain formally the Stochastic Dynamic Programming Model (also called a Markov Decision Process). Assume a system makes transitions among a finite set of states, $\mathscr{S}$. There are decision epochs, or *stages*, $n = 1,2,...N$. At each stage a decision or action a is chosen from a set of *feasible actions,* $\mathscr{A}_n(i)$. In the next stage, the system evolves to a new state j in the set of *reachable states,* $\mathscr{S}_n(i,a) \subseteq \mathscr{S}$, according to probabilities $p_{ijn}(a)$. Further assume that whenever action a is chosen in state i, the system incurs in a cost per transition of $c_n(i,a)$. The objective of the model is to choose a *policy*, that is, a set of decision rules that prescribe the action to be chosen at each stage n depending on the current state i, so that the total cost is minimized. We will denote $v_n(t)$ the value function, that is, the total expected cost from stage n onwards, if the system is in state i. Then, Bellman Principle of Optimality states that the optimal value function, $v_n^*(i)$, satisfies the equation

$$v_n^*(i) = \min_{a \in \mathscr{A}_n(i)} \left\{ c_n(i,a) + \sum_{j \in \mathscr{S}_n(i,a)} p_{ijn}(a) v_{n+1}^*(j) \right\}.$$

If the evolution of the system is deterministic, then there is a function, $f_n(a)$, called the transition function, that determines the next state. In this case the Bellman equations above reduces to

$$v_n^*(i) = \min_{a \in \mathscr{A}_n(i)} \left\{ c_n(i,a) + v_{n+1}^*(f_n(i)) \right\}$$

Bellman's equation gives us an algorithm to solve the optimal value function.: We start from a given value function for the last stage, $v_N(i)$, and proceed backwards to find all other optimal value functions, $v_n^*(i)$, for stages, *N-1*, *N-2*, etc. The optimal policy is the sequence of actions that at each stage minimize the Bellman equation.

Once these elements have been identified, a dynamic programming model has been fully defined, and it can be solved in different ways. In the following sections we defined the elements of our particular DP model to better structure a Mortgage-Backed Security, and introduce the jMDP framework to solve it [2].

B. The JMDP Framework

From the previous section it should be clear that a dynamic programming model is completely defined by determining the stages, states, feasible actions, and transition probabilities and immediate costs. jMDP is an Object Oriented framework that allows the user to build a computational representation of these mathematical objects [2][6]. By exploiting the abstraction capabilities of JAVA [9], the user does not have to worry about the algorithms needed to solve the problem, and can rather concentrate in representing the model by extending the classes provided by the framework and implementing abstract functions for the costs, probabilities, etc.

C. A Dynamic Programming Model for Mortgage-Backed Securities

To solve the MBS problem we built a Dynamic Programming model based on Cornuejols and Tütüncü [1]. It is important to keep in mind the objective function of the problem addressed in this project. As mentioned in the introduction, we seek to find an optimal structure for the Mortgage-Backed Securities, so that it would be cost efficient, that is, we seek to minimize the present value of the payments made to the bondholders by identifying this optimal structure.

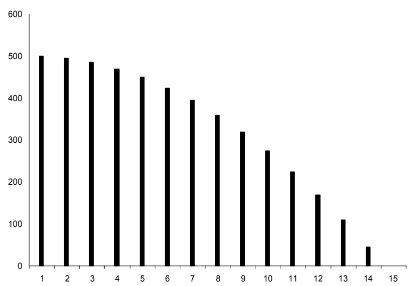

Figure 1. Annual Cash Flows

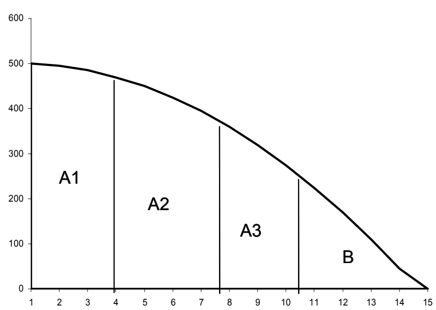

Figure 2. Cash Flows divided by tranches

Figures 1 and 2 show the payments made to the bondholders in the future, first as annual cash flows in different moments of time, and then as an area that can be divided into different tranches of payments. These tranches are the same tranches mentioned in Section II, and are paid in a sequential manner. The objective of this project is to shift from a predetermined structure of the tranches (A_1 5 years, A_2 10 years, A_3 15 years and B 15 years), to a more flexible structure where the size and the duration of each tranche is chosen in an optimal way.

The dynamic programming model that suits the conditions of this problem would be determined by the following elements:

Objective Function: Find the optimal partition of the time horizon of the issuance in 4 tranches so it would minimize the cost of issuing a MBS, by minimizing the present value of the payments made to the bondholders.

States: Years t of the time horizon of the issuance, where $t=1, 2, \ldots, 15$.

Stages: Number of tranches desired up to year T. (Four tranches: A_1, A_2, A_3, and B).

Actions/Decisions: Time length of each tranche. Say we are in state $i=t$ (initial year of the tranche), by choosing state j (final year of the tranche we would have defined the n-th tranche(i,j), and would have taken the action j (go to year j).

Transition Function: Is the function that returns the state reached once action a is taken. In this particular model, taking action a is the same as going to state a, then the transition function $f_n(a) = a$.

Immediate Cost: This cost can be found in an already constructed matrix M_{ia} that has as its elements the present value of the future payments to the bondholders of each possible tranche. The construction of this matrix was done by programming an algorithm in Java presented by Cornuejols and Tütüncü [1] that takes into account the financial variables that affect the value of issuing each possible tranche.

This algorithm is a sequential construction of matrixes where different financial variables are calculated First, we calculate the Weighted Average Life of each tranche. Then we continue to calculate the buffer of the tranches, followed by the calculation of its loss multiple. Once we know the loss multiple of each tranche we are able to determine the risk credit rating and the coupon (interest rate) of the tranches. After reaching this stage, we calculate the payments made each year to the bondholders and then we find the present value of such payments, having as outcome the matrix M_{ia} (the specific explanation of these variables was out of the immediate scope of this paper, for details see [1]).

This matrix is the main input to find the immediate cost of taking action a at a particular state i.

The dynamic programming model and the optimization process can be better understood if seen in a graphic way.

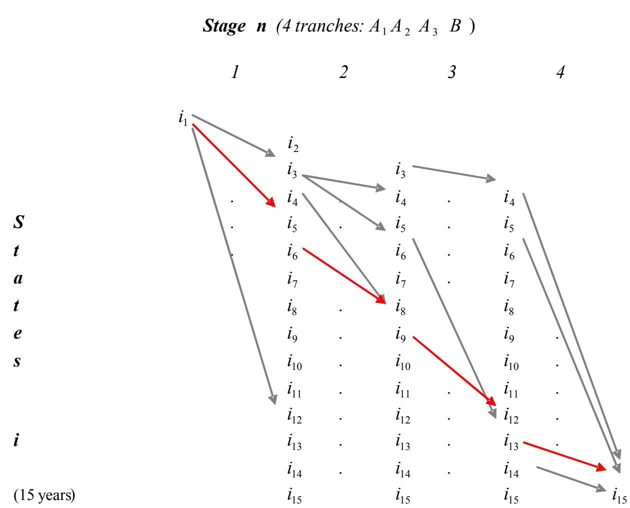

Figure 3. Graph of the DP feasible actions, 4 stages, 15 states

The graphic above shows the different feasible actions that can be taken when at state i to go from stage n to stage *n+1*. The optimal solution would show the tranches selected and its size, so we would be able to know the cost of the solution.

IV. Results

The development and implementation of the aforementioned models, had as an objective to return an optimal structure of the Mortgage-Backed Securities (TIPS) issued in Colombia by Titularizadora Colombiana, structure that would be cost efficient.

The optimization was the result of solving a dynamic programming model, and the outcome was in the form of a new, flexible structure that would fit the inner characteristics of the pool of mortgages being securitized (in particular, the seventh issuance of Titularizadora Colombiana).

The dynamic programming model found the optimal division of the issuance's time horizon, so four new tranches would be created with its particular size (amount of money underlying each tranche), risk rating, interest rate and duration (expiration date).

The model would return the total cost of the new structure, key variable in order to compare the level of improvement of the new structure with that of the current structure used to issue Colombian MBSs (TIPS). The cost of the issuance was measured in terms of the net present value of the future cash flows generated to the bondholders, which allowed us to have a net amount for both of the structures to be compared.

When extending the DP solver in the jMDP framework, we obtained the following outcomes.

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*** Best Policy (starting in Year 0 )***
Stage: 0 State: Year 0 Take Action: 2 years
Stage: 1 State: Year 2 Take Action: 4 years
Stage: 2 State: Year 4 Take Action: 8 years
Stage: 3 State: Year 8 Take Action: 15 years


Value Function:
YEAR 0       : 2268870539.73
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Figure 4. jMDP outcome (4 tranches/stages, 15 years/states)

The optimal structure found by our DP model suggested we should structure the MBS analyzed in four tranches. The first one (TIPS A_1) with a duration of 2 years, the second one (TIPS A_2) with a duration of 4 years and payments starting at the beginning of year 3, the next one (TIPS A_3) with a duration of 8 years and payments starting at the beginning of year 5, and the last one (TIPS B) with a duration of 15 years and payments starting at the beginning of year 9.

The cost of this new structure was found to be the equivalent of USD$157,161,981. To compare this cost with the current cost of the issuance, we priced the future cash flows of the seventh issuance of Titularizadora Colombiana as stated in the issuing contract, and found a net present value of USD$168,223,331. By comparing these net values we found a reduction in the cost of USD$11,061,350 or a 6.57% from the contractual cost of the issuance, making it evident that the new structure is less expensive and at the same time it is able to fulfill with all its liabilities to the bondholders. The structure will not fall in default, meaning it is a cost efficient structure.

After reaching this result, we tested the model with some sensitivity analysis, by introducing some constraints that reflect current legal constraints to the issuance of tranches with duration smaller to 5 years. We also eliminated the number tranches constraint, that is, we let the model choose the number of tranches optimal to the whole structure, and we found different improvement percentages ranging from 0.6% to 9.48% from the contractual cost, what in dollars is a cost reduction ranging from $1 to $15 million dollars, depending on the constraints introduced or taken off to the model.

These results confirm the initial hypothesis which stated that a less rigid and predetermined structure, that is, a more flexible structure that is aware of the characteristics of the pool of mortgages being securitized, will be less expensive and cost efficient.

V. Conclusions

Being mortgage securitization a newly introduced financial instrument to the capital market in Colombia, its progressive improvement is of key value to the development not only of the instrument itself but more important, to the development of the mortgage and construction sectors in the country.

By making it less expensive to issue a Mortgage-Backed Security into the market, we are opening a possibility of augmenting the number of issuances made during a year, which in turn, translate into the possibility of releasing more cash flows from the banks that can be addressed to new clients for mortgages.

The improvement and deepening of the mortgage sector is a key variable to the improvement of our emergent economy as a whole.

The model developed, based on a dynamic programming methodology, made it evident that there are still a lot of processes in which improvement can be achieved, and in particular, our model contributed to the reduction in the cost of issuing a Mortgage-Backed Security.

By shifting from a rigid, predetermined and standardized structure, to one flexible and aware of the characteristics of the pool of mortgages securitized, it was possible to reach an structure up to 9% less expensive compared to the current structure.

The implementation of a real life instance problem, by extending a dynamic programming model from the jMDP framework, proved the capacities and suitability of this framework to address Markov Decision Problems.

The development of this project in a rigorous way proved to be efficient and helped us achieve our main objectives.

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